ECE 443/643 Homework 3

October 21, 2011

1. A Hilbert transformer is an LTI system with frequency response $H(f) = -j\text{sgn}(f)$. The Hilbert transform of a signal $\mathcal{H}\{x(t)\} = \hat{x}(t)$ is the output of the Hilbert transformer when $x(t)$ is input into it.
   (a) Is the Hilbert transformer a real system? Justify your answer (e.g. show that any real input gives a real output, provide a counterexample, etc.).
   (b) Is the Hilbert transformer causal? Justify your answer.
   (c) Is the Hilbert transformer BIBO stable? Justify your answer.

2. Let $\mathcal{H}\{x(t)\} = \hat{x}(t)$ be the Hilbert transform of an energy signal.
   (a) Show that the energy spectral density (ESD) of $\hat{x}(t)$ is equal to the energy spectral density of $x(t)$.
   (b) Show that the energy inner product $\langle \hat{x}, x \rangle = 0$ if $x(t)$ is real.

3. Let $x(t) = \sum_{n=-\infty}^{\infty} X[n] e^{2\pi j f_0 n t}$ be a periodic power signal. If $x(t)$ is input into a Hilbert transformer,
   (a) Find spectrum $\hat{X}[n]$ of the output in terms of the input.
   (b) Find the power spectral density (PSD) of the input.
   (c) Find the PSD of the output.

4. Let $x(t) = A_c(1 + \mu \sin(2\pi f_m t)) \cos(2\pi f_c t) = \Re(\tilde{x}(t) e^{2\pi j f_c t})$. Assume $f_m \ll f_c$.
   (a) Sketch $x(t)$ for $\mu = 0, \frac{1}{2}, 1, \frac{3}{2}$.
   (b) Sketch the output of an ideal envelope detector with $x(t)$ as input for $\mu = 0, \frac{1}{2}, 1, \frac{3}{2}$.

5. Let $x(t) = \Re(\tilde{x}(t) e^{2\pi j f_c t})$ and $m(t) = \text{sinc}(t)$. *Note: the tilde doesn't have any special meaning, but the circumflex (hat) means Hilbert transform, as in problem 1.*
   (a) If $\tilde{x}(t) = m(t)$, find $\tilde{X}(f)$ and $X(f)$.
   (b) If $\tilde{x}(t) = m(t)$, sketch $\tilde{X}(f)$ and $X(f)$.
   (c) If $\tilde{x}(t) = m(t) - j\hat{m}(t)$, find $\tilde{X}(f)$ and $X(f)$.
   (d) If $\tilde{x}(t) = m(t) - j\hat{m}(t)$, sketch $\tilde{X}(f)$ and $X(f)$.
   (e) If $\tilde{x}(t) = m(t) + j\hat{m}(t)$, find $\tilde{X}(f)$ and $X(f)$.
   (f) If $\tilde{x}(t) = m(t) + j\hat{m}(t)$, sketch $\tilde{X}(f)$ and $X(f)$.